Price and Quantity Solution Method for Tenders

White Paper, 22 June 2016 (Version 1.0)

INTRODUCTION

GDT Marketplace has two types of trading model: Fixed Price and Tender. In the Fixed Price model, the person putting up the listing specifies the sale price and the product is sold to buyers in the order in which they submit purchase requests until the available quantity is sold. In the Tender model, both the sale price and the allocation of quantity between buyers is determined by a software algorithm.

This White Paper provides a high level description of the solution method that GDT uses for the Tender model.

The paper shows how the Tender solution achieves an efficient outcome in the presence of both single- and multi-bid buyers, and also in response to 'lumpy' buy or sell requirements.

Lumpy requirements are situations where a buyer or seller has specified a large minimum quantity that either must be achieved or the person prefers to not trade at all.

For example, a seller may list 100 MT of a product for Tender which attracts two buyers to bid for 50 MT each at different prices while a third 'lumpy bidder' bids for the entire 100 MT at a price that lies between the other two bids. The lumpy bid forces a choice between either allocating all 100 MT to the single large bid or allocating 50 MT to each of the two smaller bids.

The solution method outlined below shows how this choice is made and explains how winning prices are determined.

DESIGN OBJECTIVES

Three key objectives guided the design of the GDT Marketplace Tender system for determining winning prices and quantities:

1. Efficiently maximise total value gain across buyers and sellers;
2. Simplify the decision processes faced by sellers and buyers;
3. Enable buyers and sellers to reflect their degree of flexibility over the quantities they may be allocated.

The first objective implies that the Tender solution should sell as much as possible of the available supply and to do so by allocating to the bidders with the highest willingness to pay relative to the seller's reserve price.

The second objective of minimising decision complexity has several important implications:

- The seller's reserve price should be private and not observable to buyers;
- Wherever possible the winning price paid by a successful bidder should be independent of the price they bid, so that buyers can set their bid prices based on what they are willing to pay without the complexity of needing to correctly guess how other buyers are likely to bid.

The third objective of enabling buyers and sellers to reflect their degree of flexibility implies allowing them to set upper and lower limits on the quantities acceptable to them, as explained below.

BUYER AND SELLER REQUIREMENTS

The Tender model allows the seller and each buyer to specify the range of quantities that meet their requirements, as follows:

Seller Inputs

The seller specifies four quantity and pricing requirements when setting up a Tender listing:

a. The maximum quantity available for sale (called the seller's offer quantity);
b. The minimum total quantity that must be sold or otherwise the seller would prefer to not sell anything (minimum offer quantity);

c. The minimum parcel size that must be sold to each successful buyer (minimum bid quantity);

d. The minimum price below which a bid must be rejected (reserve price).

For example, a seller who has flexibility in how much is sold and is willing to ship to multiple buyers could specify offer quantity at 100 MT, minimum offer quantity at 15 MT (being one full container load), and minimum bid quantity at 15 MT (being a full container load). This would enable any quantity between 15 – 100 MT to be sold in lots of full container loads.

Alternatively, a seller who is highly inflexible and simply wishes to sell 100 MT in a single lot to the highest bidder would specify each of the offer quantity, minimum offer quantity, and minimum bid quantity at 100 MT. This indicates that exactly 100 MT must be sold and requires each buyer to submit bids for exactly 100 MT.

Buyer Inputs

Each buyer who bids on a Tender listing specifies the following requirements:

a. The maximum quantity the buyer is willing to buy at or below a specified price (buyers can enter multiple price, quantity bids);

b. The minimum acceptable quantity that the buyer requires or otherwise he or she prefers to not buy anything.

For example, a buyer who wishes to purchase 60 MT up to a certain price but is willing to accept smaller quantities could specify maximum quantity at 60 MT and minimum acceptable quantity at 15 MT (being one full container load). This would enable the buyer to win any quantity between 15 – 60 MT.

Alternatively, a buyer who is highly inflexible and wishes to purchase either the full 60 MT or nothing would specify maximum quantity and minimum acceptable quantity at 60 MT. This indicates that the buyer must be allocated either exactly 60 MT or zero.

Hence, either or both of the buyer and seller may be flexible or inflexible. A large inflexible quantity is referred to as ‘lumpy’.

SOLUTION METHOD

Given the design objectives outlined above, the Tender solution method implemented for GDT Marketplace seeks to achieve three outcomes:

i. Efficiently maximise the total value gain across buyers and the seller;

ii. Wherever possible, ensure all successful buyers pay the same price (uniform pricing); and

iii. Achieve an average selling price as close as possible to what would have occurred if all bids were fully flexible.

The solution algorithm first checks whether the listing is under-subscribed. This occurs if the aggregate bid quantity at prices equal or above the reserve price is less than the seller’s minimum offer quantity. In this situation, no quantity is sold and the Tender solution process has finished.

Assuming the listing is not undersubscribed, the solution method is consistent with the following three step process:

Step 1: Set the ‘target price’ assuming no minimum bid quantities

Derive the ‘target price’ that maximises the total value gain across buyers and seller subject to the constraints that:

- No buyer would pay more than its bid price for the quantity allocated to it;
- No buyer would be allocated more than its bid quantity at the price it pays;
- Total sold quantity would be set at the lesser of the aggregate bid quantity and the seller’s offer quantity.

Step 2: Set the winning quantities consistent with minimum bid constraints

Choose the set of winning quantities across buyers to maximise total value gain subject to:

- No buyer wins more quantity than its bid quantity;
- No successful buyer wins less than its minimum acceptable quantity;

- Total sold quantity falls within the seller's supply range (i.e. between offer quantity and minimum offer quantity);

In the event that more than one solution is possible, the Tender will, in effect, make a random selection between the possible solutions.

Step 3: Set winning prices as close as possible to target price

Taking as given the target price from Step 1 and the winning quantities from Step 2, set the winning prices across bidders to minimise the variability of winning prices from target price, subject to:

- Each successful buyer pays a single winning price which is does not exceed its bid price for the quantity allocated to it;
- The quantity-weighted average price received by the seller is as close as possible to the target price.

EXAMPLE ONE

This section shows how the solution method outlined above applies to the simple example in the introduction where a Tender listing for 100 MT has attracted two bids for 50 MT each and a third bid for the entire 100 MT at a price that lies between the other two bids.

The seller is assumed to be fully flexible and accordingly sets its minimum offer quantity and minimum bid quantity at zero. Reserve price is $1,800 per MT.

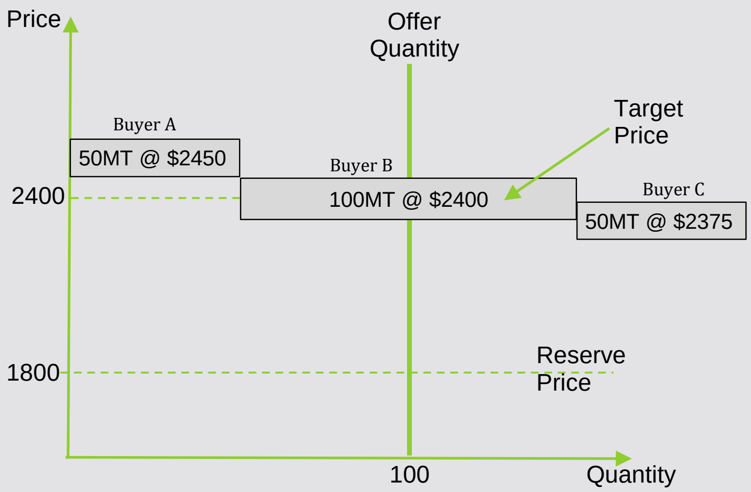
In this example, Buyer A has bid for 50 MT at a maximum price of $2450, Buyer B has bid for 100 MT at $2400 and Buyer C has bid for 50 MT at $2375.

The three step process is as follows:

Step 1: Set the 'target price' assuming no minimum bid quantities

Arranging the bids in price order, the target price is derived where aggregate bid quantity crosses with the offer quantity. In this case, target price is $2400.

Figure 1: Determination of Target Price



Step 2: Set the winning quantities consistent with minimum bid constraints

Case A: All buyers fully flexible

If all buyers are fully flexible with minimum acceptable quantities set to zero, then the optimal outcome is a straight forward allocation of winning quantity to the buyers with the highest willingness to pay.

Buyer A would receive its full bid of 50 MT, Buyer B would also receive 50 MT (half its bid quantity), and Buyer C would receive nothing.

Value gain is measured as the difference between the successful buyers' marginal value versus the seller's marginal opportunity cost, both multiplied by and summed over sold quantities.

In this example, the value gain is the sum of (Bid Price – Reserve price) x Sold Quantity:

Total value gain = ($2450 – $1800).50 + ($2400 – $1800).50 = $62,500.

Case B: Lumpy bidder

Now suppose that Buyer B is inflexible and has specified a minimum acceptable quantity equal to 100 MT so that he or she wins either 'all or nothing'. Buyer B is a lumpy bidder.

The presence of the lumpy bid forces a choice between either allocating all 100 MT to Buyer B or allocating 50 MT to each of Buyers A and C.

In this example, selling to Buyers A and C would generate $61,250 of value whereas selling to Buyer B would generate only $60,000 of value. Hence, Buyers A and C would receive their full bid quantities and the lumpy bidder, Buyer B, would receive nothing.

Option 1: Allocate 50 MT to each of Buyers A and C:

Total value gain = ($2450 – $1800).50 + ($2375 – $1800).50 = $61,250

Option 2: Allocate 100 MT to Buyer B:

Total value gain = ($2400 – $1800).100 = $60,000.

Case C: Different bid prices

The above allocation reflects the relative value of the prices bid by the lumpy bidder versus the flexible bidders.

If either Buyer B had bid a sufficiently higher price or one or both of Buyers A and C had bid sufficiently lower prices, then the optimal allocation would switch in favour of the lumpy bidder.

For example, if Buyer C had bid at $2300 (rather than $2375) then the value gain of selling to Buyers A and C would be only $57,500, which is less than the $60,000 value gain of selling to Buyer B. Hence, Buyer B would be allocated 100 MT and Buyers A and C would win nothing.

Option 1: Allocate 50 MT to each of Buyers A and C:

Total value gain = ($2450 – $1800).50 + ($2300 – $1800).50 = $57,500

Option 2: Allocate 100 MT to Buyer B:

Total value gain = ($2400 – $1800).100 = $60,000.

Step 3: Set winning prices as close as possible to target price

From Step 1 the target price is $2400. Wherever possible, the aim is to achieve uniform pricing where the winning price for each successful buyer is set equal to the target price.

Uniform pricing can be achieved in Cases A and C because in both cases all successful bids were bid at or above the target price:

Case A: Buyers A and B win 50 MT each

Buyer A specified a bid price of $2450, whereas Buyer B specified a bid price of $2400. Both are equal or higher than the target price of $2400. Hence, both pay $2400.

Case C: Lumpy bidder wins 100 MT

Buyer B is the only winning bidder and was in fact the marginal bidder who determined the target price. Hence, Buyer B pays his or her bid price of $2400.

Uniform pricing cannot be achieved when one or more of the successful bids were bid at below the target price. The algorithm minimises the variance of winning prices from target price while also seeking to achieve an average selling price as close as possible to the target price. This is illustrated in Case B:

Case B: Buyers A and C win 50 MT each

Buyer A specified a bid price of $2450, whereas Buyer C specified a bid price of $2375, which is less than the target price of $2400.

The best that can be achieved is for Buyer C to pay $2375 (his or her maximum bid price) and to make up the shortfall by setting Buyer A's winning price at $2425, to give an average selling price equal to the target price of $2400.

Note that although Buyer A's winning price of $2425 is higher than in Case A, it is still lower than Buyer A's bid price of $2450. Buyer A is still paying less than the maximum price specified in his or her bid.

EXAMPLE TWO

This section illustrates the solution algorithm where one or more buyers enter multiple price/quantity bids.

In this example, instead of two small buyers (A and C) we assume that Buyer A enters two bids, one for 50 MT at $2450 and a second bid for 100 MT at $2400. We continue with the large Buyer B who wishes to purchase 100 MT and the seller supply parameters are also unchanged from the previous example.

The three step process is as follows:

Step 1: Set the 'target price' assuming no minimum bid quantities

Arranging Buyer A's two bids appropriately, the target price is $2400.

Figure 2: Determination of Target Price

Step 2: Set the winning quantities consistent with minimum bid constraints

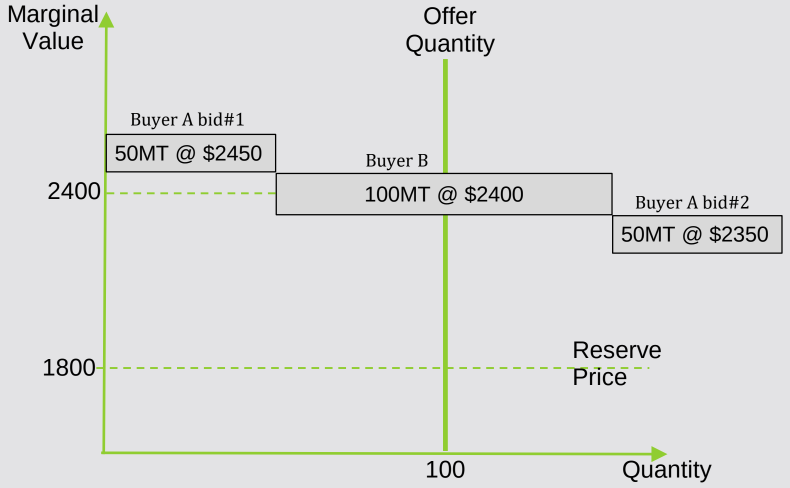
In any situation where one or more buyers has submitted multiple bids, the allocation of winning quantities is determined by converting the bids to marginal value terms.

For Buyer A, bid #1 is the highest-price bid and its marginal value is given by the bid price, $2450. For bid #2, the marginal value of the additional 50 MT requested is given by the incremental amount that Buyer A is prepared to pay divided by the additional quantity:

Marginal value of additional 50 MT in bid #2 = (2400 x 100 – 2450 x 50)/50 = $2350.

Figure 3 below illustrates the marginal value curve:

Figure 3: Marginal value curve



Once again, the allocation depends on whether Buyer B is flexible to receive less than 100 MT or has submitted a lumpy bid for exactly 100 MT:

- If Buyer B is flexible, then the first 50 MT is allocated to Buyer A bid #1 and the remaining 50 MT is allocated to Buyer B;
- If Buyer B is inflexible, then the full 100 MT could be allocated to either Buyer A or Buyer B as in this example the total value gain is identical. The Tender will, in effect, make a random selection these two possible outcomes.

  Option 1: Allocate 50 MT to Buyer A bid #1 and further 50 MT to Buyer A bid #2

  Total value gain = ($2450 – $1800).50 + ($2350 – $1800).50 = $60,000

  Option 2: Sell 100 MT to Buyer B:

  Total value gain = ($2400 – $1800).100 = $60,000.

Step 3: Set winning prices as close as possible to target price

Each successful buyer always pays a single winning price which does not exceed the relevant bid price for the quantity allocated to the buyer.

In this example, since the bid prices in all successful bids are at or above the target price, each buyer pays the same winning price of $2400.

CONCLUSIONS

The solution method outlined above has three main features:

1. The target price is determined by comparing demand versus supply, using all bids irrespective of whether they have lumpy requirements;
2. The allocation of winning quantities to bidders is determined so as to maximise the total value gain across buyers and the seller;
3. Winning prices are derived to minimise deviations from target price while seeking wherever possible to achieve an average selling price equal to the target price.

These features provide a robust method for determining Tender results that achieves an efficient outcome with either single-bids or multi-bids and in the presence of lumpy buy or sell requirements.